The Alternating Series Remainder Theorem

Next, we have the **Alternating Series Remainder Theorem**. This is the favorite remainder theorem on the AP exam! The theorem tells us that if we take the sum of only the first n terms of a converging alternating series, then **the absolute value of the remainder of the sum (the terms we left off) will be less than or equal to the value of the first term we left off.** In other words, if we take the partial sum S_5 (the sum of the first 5 terms: $a_1 + a_2 - a_3 + a_4 - a_5$), then the remainder will be less than the value of a_6 and have the same sign as that term.

This makes sense. Since the terms alternate in sign and are decreasing in size, the first term we leave off the sum will have the largest absolute value of any other of the missing terms. So the error has to be less than or equal to that term.

Note: in order to use this you have to first show that you have a convergent alternating series! ** This is always on the AP exam - remember it.

Here is an example. I will do more in the HW examples.

E: Approximate the sum of the series with the first 6 terms of the series. Then give the error of the sum and the range of the actual sum.

This means that we first show the series is convergent, then find the sum of the first 6 terms, and finally find the error (remainder) to give a range of the actual sum.

$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n!}$	This is our series.
$\lim_{n\to\infty}\frac{1}{n!}=0$ the terms approach 0 $\frac{1}{(n+1)!}<\frac{1}{n!}$ the terms decrease in size The series converges by the AST.	First I have to show that the series is convergent. ** We don't have to show absolute convergence, so just use the AST.
$S_6=1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24}+\frac{1}{120}-\frac{1}{720}\approx 0.63194$	Find the approximate sum by adding the first 6 terms.
$\lvert R\rvert \le a_7=\frac{1}{5040}\approx 0.0002$	Now find the value of the remainder of the series.
$0.63194 - 0.0002 \le S_6 \le 0.63194 + 0.0002$ $0.63174 \le S_6 \le 0.63214$	Thus our actual sum will be between $S_6 \pm R$